13/4/2023

~~Units and Measurement~~

Mathematical Tools

$(a+b)^2 = a^2 + b^2 + 2ab$
$(a-b)^2 = a^2 + b^2 - 2ab$
$a^2 - b^2 = (a+b)(a-b)$
$(a+b)^3 = a^3 + b^3 + 3ab(a+b)$
$(a-b)^3 = a^3 - b^3 - 3ab(a-b)$

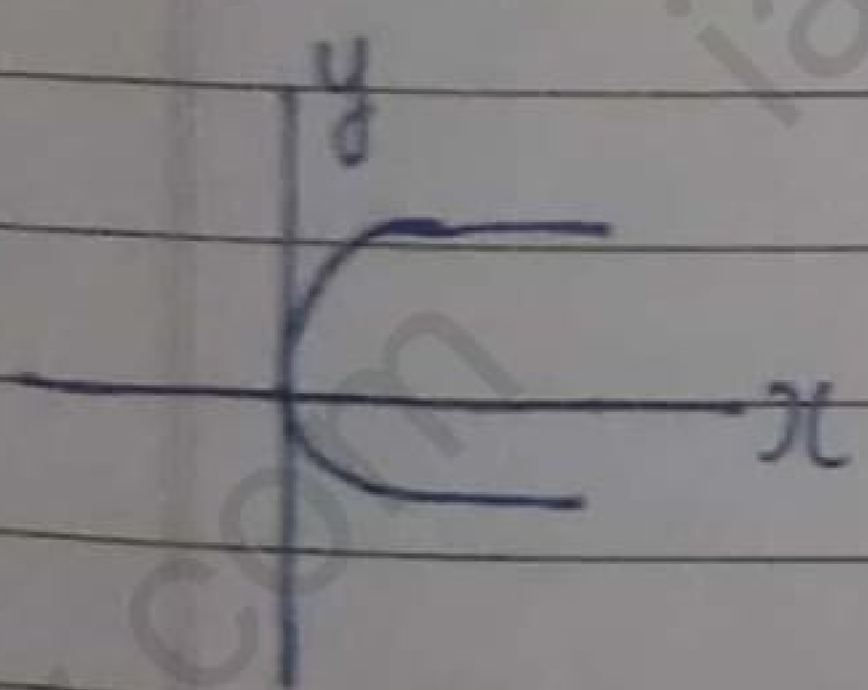
Binomial theorem

$x << 1$

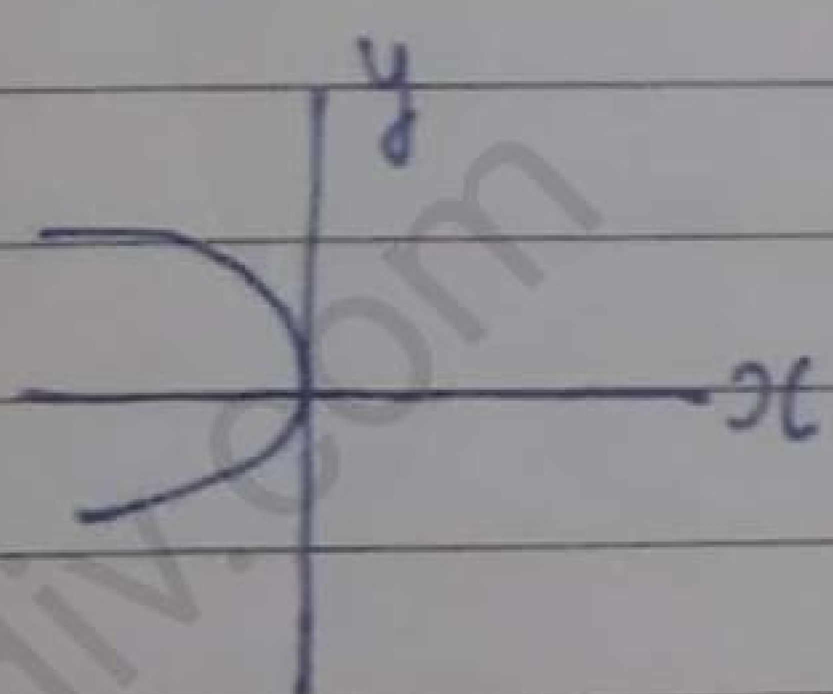
$(1+x)^n = 1 + nx$

Parabolic Equation

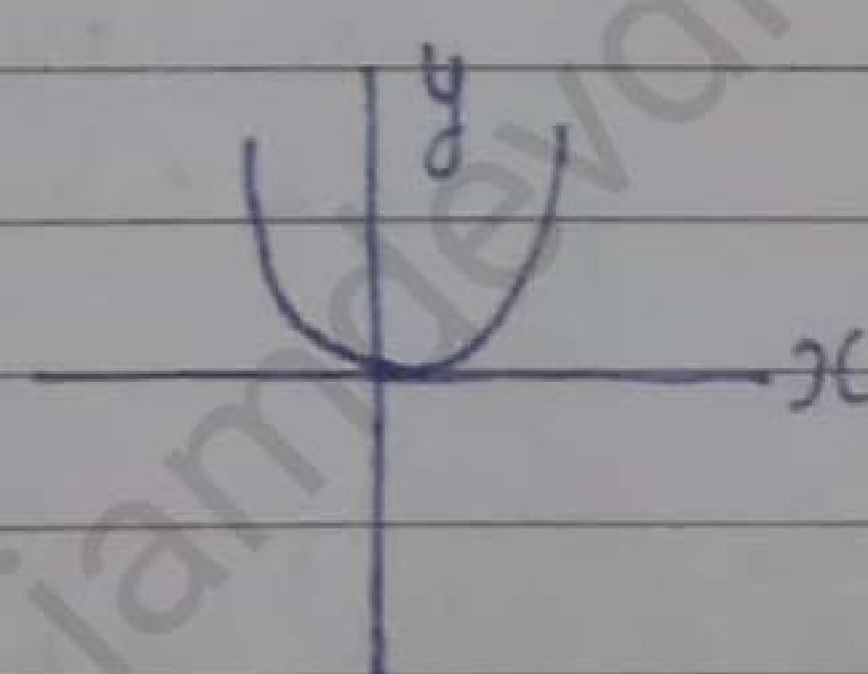
$y^2 = Kx$



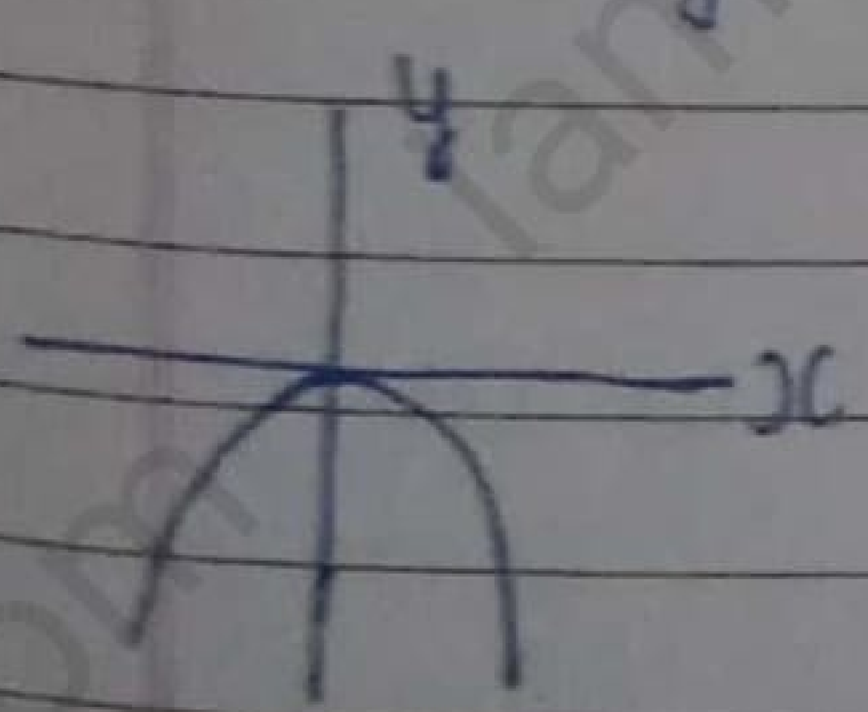
$y^2 = -Kx$



$x^2 = Ky$

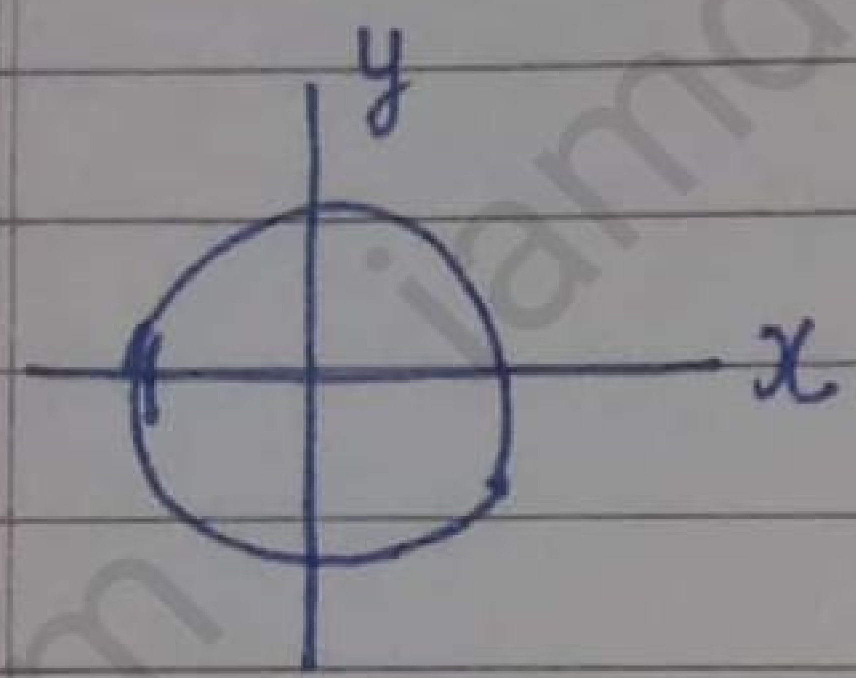


$x^2 = -Ky$



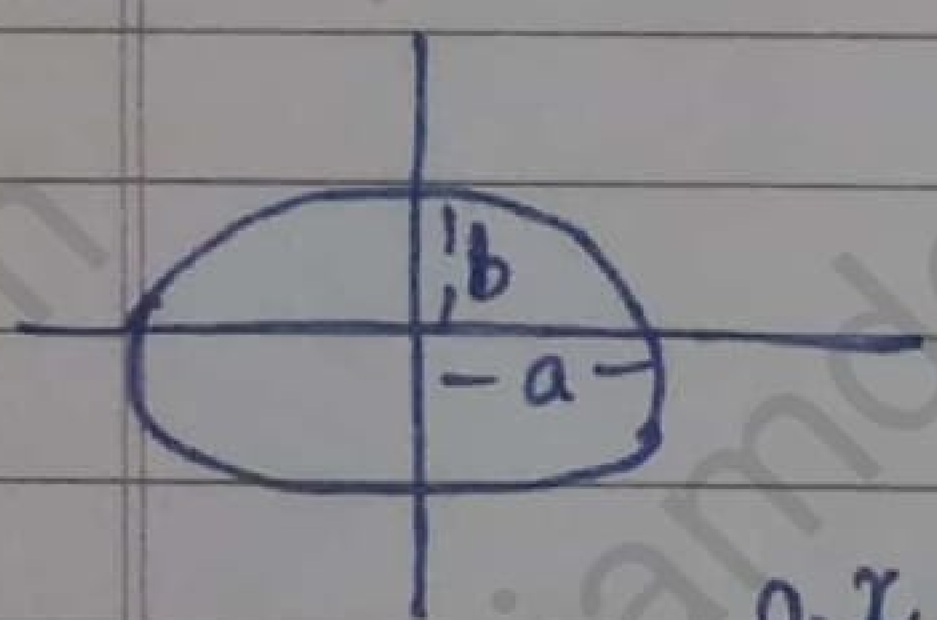
Circle - 2D

$x^2 + y^2 = a^2$



Ellipse

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$



$a x \neq a y$

$a x > a y$

$a x$ - major axis

$\sin^2\theta + \cos^2\theta = 1$

$\sin 2\theta = 2\sin\theta\cos\theta$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$\cos^2\theta - (1 - \cos^2\theta)$

$\cos^2\theta = 2\cos^2\theta - 1$

$\cos^2\theta = (1 - \sin^2\theta) - \sin^2\theta$

$\cos\theta = 1 - 2\sin^2\theta$

$\log_{10}{mn} = \log_{10}{m} + \log_{10}{n}$

$\log_{10}{\frac{m}{n}} = \log_{10}{m} - \log_{10}{n}$

~~$\log_{10}{m}$~~ $\log_{10}{m^n} = n \log_{10}{m}$

Large to small

$\frac{d}{dx} x^n = n x^{n-1}$

$v = \frac{\Delta x}{\Delta t}$

$= \frac{x_2 - x_1}{t_2 - t_1}$

$\frac{d}{dx} x^3 = 3x^2$

$\frac{d}{dx} x^9 \Rightarrow \frac{d}{dx}\left(\frac{1}{x^2}\right)$

$\Rightarrow \frac{d}{dx} x^{-2}$

$= -2x^{-2-1} \Rightarrow -2x^{-3}$

Ex:- $\bar{x} = 3t^2$

$t = 2sec$

$v = \frac{d}{dt} x$

$= \frac{d}{dt} (3t)^2$

$3 \frac{dt^2}{dt} = 3 \times 1t^2 \Rightarrow 6t$

$\Rightarrow 6 \times 2$

$\Rightarrow 12m^2$

* $x = 4t^3$
  $v = 12t^2$

* $x = t^9$

$$\frac{dx}{dt} = \frac{d}{dt} t^9$$

$$a = \frac{d}{dt} v$$

$x \rightarrow v \rightarrow a$

$\int$ - Integration

$\oint$ - Close Integration

$\iint$ - Surface Integration or $\int_S$

$\iiint$ or $\int_V$ - Volume Integration

$$\int x^n dx = \frac{x^{n+1}}{n+1}$$

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Ex $\int x^6 dx = \frac{x^7}{7}$

Science
↓
A - Arabic - ILM
S - Sanskrit - Vijnan
L - Latin → To know (Scientic)

Physics
↓
fusis (Greek word)
↳ Nature
Nature & natural phenomenon

A Beautiful combination of Philosphy and mathematical Science

18/April/2023

Chapter-1

Units, Dimensions & Error Analysis

All the quantities which are used to describe the laws of physics are called physical quantities.
Ex:- length, mass, volume, etc.

- To express the ~~measu~~ measurement of a physical quantity we need to know two thing as given below

(i) The unit in which the quantity is measured.
(ii) The numerical value or the magnitude of the quantity.

i.e. The no. of times that unit is contained it in the given physical quantity $= nu$

$$n \propto \frac{1}{u} \Rightarrow nu = \text{constant}$$

→ Here, n = numerical value of the physical quantity and u = Size of unit

We may also write it as:-

$$\boxed{n_1 u_1 = n_2 u_2}$$

Where n_1 and n_2 are values of the physical quantity in two different units u_1 and u_2

→ The standard amount of a physical quantity chosen to measure the physical quantity of same kind is called a physical unit.

- The essential requirements of physical unit are given below:-

(i) It should be of suitable size
(ii) It should be easily accessible.
(iii) It should not vary with time.
(iv) It should be easily reproducible.
(v) It should not depend on physical conditions like pressure volume, etc

System of units

A complete set of units which is used to measure all kinds of fundamental & derived quantities is called a system of units.

(i) CGS system:- In this system, the units of length, mass and time are cm, gram(g) and second (s) respectively. The unit of force in this system is dyne and that of work or energy is erg.

(ii) FPS System - In this system, the units of length, mass and time are foot (ft) pound (lb) and second (s) respectively. The unit of force in the system is poundal.

(iii) MKS system - In this system, the units of length, mass and time are (m), (kg), (s) respectively. The unit of force in this system is newton (N) and that of work or energy is Joule (J)

(iv) International system (SI):- This system of units helps in revolutionary changes over the MKS system and is known as rationalised MKS system. It is helpful to obtain all the physical quantities in physics.

Note:-

(i) The FPS system is not a metric system. This system is not in much use these days.

(ii) The drawback of CGS system is that many of the derived units on this system are inconveniently small.

* Fundamental quantities and fundamental units

Those physical quantities which are independent of each other and not defined in terms of other physical quantities are called fundamental quantities or base quantities. The units of these quantities are called fundamental or base units.

* Derived quantities and derived units

The quantities which can be expressed in terms of the fundamental quantities are called derived quantities. The units of these quantities are called derived units.

eg. Unit of speed = ms^{-1} can be derived from fundamental units, i.e. unit of length and time as

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}} = \frac{m}{s} = ms^{-1}$$

* Supplementary quantities and supplementary units

Other than fundamental and derived quantities, there are two or more quantities called as supplementary quantities. The units of these quantities are known as supplementary units

Dimension :-

Dimension of a physical quantity are the powers to which the units of base quantity are raised to represent a derived unit of that quantity.

The expression which show how & which of the base quantities represent a physical quantity is called the dimension formula of given physical quantity.

when a physical quantity is equated to its dimensonal formula, what we obtained is the dimensial epciation of physical quantity

	Physical Quantity	Unit	Dimensional formula
1.	Length	Metre	[L]
2.	Time	second	[T]
3.	Mass	kg	[M]
4.	Distance	Metre	[L]
5.	Displacement	Metre	[L]
6.	Area	m^2	$[L^2]$
7.	Volume	m^3	$[L^3]$
8.	velocity	ms^{-1}	$[LT^{-1}]$
9.	speed	ms^{-1}	$[LT^{-1}]$
10.	Acceleration	ms^{-2}	$[LT^{-2}]$
11.	Acceleration due to gravity	ms^{-2}	$[LT^{-2}]$
12.	Force	$kg\,ms^{-2}$	$[NL^{-2}]$

14 K.E.	~~$\frac{1}{2}mv^2$~~ Joule	$\frac{1}{2}mv^2$ $[ML^2T^{-2}]$
15 P.E.	Joule	mgh $[ML^2T^{-2}]$
16. Momentum	~~Joule~~ Mass x Velocity	$\tau = \bar{r} \times \bar{F}$ $[M^1L^1T^{-1}]$
17 Linear momentum (P) $P = mv$	kg ms^{-2}	$[MLT^{-2}]$
18. Impulse (f×t)	N.sec $= kg\ ms^{-2}$	$[MLT^{-2}]$
19. Frequency	$[sec^{-1}]$	$[T^{-1}]$
20. Gravitational constant $\frac{FR^2}{M^2}$	~~$kg\ m^3 s^{-2}$~~ $kg^{-1}m^3s^{-2}$	$[M^{-1}L^3T^{-2}]$
~~20~~21. frequency	$\frac{1}{T}$	$[M^0L^0T^{-1}]$
22. Stress	$\frac{Newton}{m^2}$	$\frac{F}{A}$ $[ML^{-1}T^{-2}]$
23. Strain	$\frac{metre}{metre}$	$\frac{\Delta L}{L}$ (change in length) $[M^0L^0T^0]$
24. Young Modulus Y,	$\frac{Stress}{Strain}$	$[ML^1T^{-2}]$
25. Impulse ~~F×Δt~~	$F \times \Delta t$	$[M^1L^1T^{-1}]$
26. Angular velocity (ω)	$\frac{\text{Angular displacement}}{\text{Time}}$	$\frac{\theta}{t} = \frac{1 (Radian)}{sec}$
~~27~~	sec^{-1} $[T^{-1}]$	$[M^0L^0T^{-2}]$
27. Angular acceleration	$R = \frac{sec^{-1}}{sec^2}$	$[T^{-2}]$
28. Angular momentum (L)	$M \times \frac{L}{T} \times L$ (mvr Sinθ)	$[ML^2T^{-1}]$
29. Moment of Inertia (I)	mr^2 (ML^2)	$[M^1L^2T^0]$ ~~MR^2~~
30. Torque ~~(F×d×Sin~~	F × d × Sinθ	$[ML^2T^{-2}]$

31	Cofficient of viscosity (η)	$n = \frac{F}{\gamma v}$	$[ML^{-1}T^{-1}]$
	$\eta = F$		
32	$PV = n R T$ – Temp (n – mol)	$\frac{PV}{nT}$	$\left[\frac{ML^{-1}T^{-2}L^3}{mol^{-1}}\right]$
33	Electrical potential	$V = \frac{W}{q}$	$[M^1L^2A^{-1}T^{-3}]$
34	Electric Resistance	$R = \frac{V}{I}$	$\left[\frac{M^1L^2A^{-1}T^{-3}}{A^1T^1}\right]$
35.	Pressure	$\frac{Force}{Area}$	$[ML^{-1}T^{-2}]$
36.	Radius of gyration	$\sqrt{\frac{I}{\Sigma m_i}}$	$[M^0L^1T^0]$
37.	Angular frequency	$2\pi v$	$[M^0L^0T^{-1}]$
38.	Angular impulse	Torque X time	$[ML^2T^{-1}]$
39.	Planck's constant	$\frac{Energy}{frequency}$ $\left[\frac{E}{v}\right]$	$[ML^2T^{-1}]$
40.	Efficiency	$\frac{Output\ work}{Input\ work}$ $\left[\frac{W}{Q}\right]$	$[M^0L^0T^0]$
41.	Heat Capacity	$\frac{Heat\ energy}{Temp.}$ $\left[\frac{Q}{T}\right]$	$[ML^2T^{-2}K^{-1}]$
42.	Specific heat capacity.	$\left[\frac{Q}{m \times \Delta T}\right]$	$[M^0L^2T^{-2}K^{-1}]$
43.	Thermal conductivity, K	$\frac{Q \times thickness}{Area \times Temp. \times time}$	$[MLT^{-3}K^{-1}]$
44.	Thermal Resistance, R	$\frac{Length}{Thermal\ conduct. \times area}$	$[M^{-1}L^{-2}T^3K]$
45.	Electric field	$\frac{F\ (force)}{q\ (charge)}$	$[MLT^{-3}A^{-1}]$

Use of Dimension Analysis

Check the correctness of given physical quantity

Conversion from one system to another system.

Establish a relation between given physical quantity.

$F = m a$

Force ↓ mass ↳ acceleration

$[MLT^{-2}] = [M][LT^{-2}]$

L.H.S. = R.H.S

Ex2 $F = M r \omega$

↓ Force ↓ Mass ↳ Angular velocity $[T^{-1}]$

$[MLT^{-3}]$ $[M]$

DLHS D R.H.S.

$[MLT^{-2}] = [MLT^{-2}]$

L.H.S. = R.H.S.

$F = \frac{mv^2}{R^2}$ Force

m → Mass

v^2 – velocity

R^2 – Radius

~~[M] [MLT^{-1}] ≠ [M^3] [L] [T]~~

$$L.H.S. = [M^1 L^1 T^{-2}]$$

$$R.H.S. = \frac{kg\, m^2\, s^{-2}}{m^2} \Rightarrow [M^1 T^{-2}]$$

$L.H.S. \neq R.H.S.$

Ex:-

$v = u + at$ → Time

v → Final velocity; u → Initial velocity; a → acc.

$$[LT^{-1}] = [LT^{-1}] + [LT^{-2}][T^1]$$
$$= [LT^{-1}] + [LT^{-1}]$$
$$[LT^{-1}] = [LT^{-2}]$$
$$D\,L.H.S. = D\,R.H.S.$$

$v^2 = u^2 + 2as$ → distance

v ↓ Final vel.; u ↓ Initial vel.; a ↓ acceleration

~~$[L^2T^{-1}]^2 = [LT^{-1}] + 2[LT^{-2}]$~~

$$[L^2T^{-2}] = [L^2T^{-2}] + 2[L^2T^{-2}]$$
$$[L^2T^{-2}] = [L^2T^{-2}]$$
$$L.H.S. = R.H.S.$$

Ex:- $S = ut + \frac{1}{2} a t^2$ → Time

↓ distance

$$[L] = [LT^{-1}][T] + \frac{1}{2}[LT^{-2}][T^2]$$

~~$= [LT^{-1}] + \frac{1}{2}[LT^{-1}]$~~

$$= [L] + \frac{1}{2}[L]$$

$$= [L] = \frac{3}{2}[L]$$

D L.H.S. = D R.H.S.

Ex:- $S_n = U_n + \frac{1}{2} a (2n-1)$ → Time

S_n ↓ distance / Time, U_n ↓ velocity

$$[LT^{-1}] = [LT^{-1}] + \frac{1}{2}[LT^{-2}][T]$$

$$= [LT^{-1}] + \frac{1}{2}[LT^{-1}]$$

$$[LT^{-1}] = \frac{3}{2}[LT^{-1}]$$

D L.H.S. = D R.H.S.

Conversion from one system to another system

Ex1:- $g = 9.8 \frac{m}{s^2} = 9.8 \; m s^{-2}$

$\downarrow$ n_1 Magnitude $\quad \downarrow$ unit u_1

$n \propto \frac{1}{u} \qquad n_1 u_1 = n_2 u_2$

$n_2 = n_1 \frac{u_1}{u_2}$

$= 9.8 \frac{m}{cm} \frac{s^{-2}}{s^{-2}} = 9.8 \left[\frac{m}{cm}\right] \left[\frac{s}{s}\right]^{-2}$

$= 9.8 \left[\frac{100 \; cm}{cm}\right]$

$n_2 = 980$

C.G.S. $g = 980 \; cm \, s^{-2}$

Ex2:- $F = 1N$

$= 1 \; kg \; m \, s^{-2}$ $\qquad 1 \; kg = 1000$

$n_2 = ? \quad u_2 = gm \; cm s^{-2}$

$n_2 = n_1 \frac{u_1}{u_2}$

$= 1 \frac{kg}{g} \frac{m}{cm} \frac{s^{-2}}{s^{-2}}$

$= 1 \left[\frac{kg}{g}\right] \left[\frac{m}{cm}\right]$

$= 1 \left[\frac{10^3\ \cancel{gm}}{\cancel{gm}}\right] \left[\frac{10^2\ \cancel{cm}}{\cancel{cm}}\right]$

$\boxed{n_2 = 10^5}$

$F = 10^5\ gm\ cm\ s^{-2}$

$\boxed{1\ N = 10^5\ dyne}$

Ex 3:- 1 Joule - $kg\ m^2 s^{-2}$

$n_2 = ?$ $\quad u_2 = g\ cm^2 s^{-2}$

$n_2 = n_1 \frac{u_1}{u_2}$

$= 1 = \frac{kg}{g} \frac{m}{cm^2} \frac{\cancel{s^{-2}}}{\cancel{s^{-2}}}$

$= 1 = \frac{kg}{g} \left[\frac{m}{cm^2}\right]$

$= \left[\frac{10^3\ \cancel{gm}}{\cancel{gm}}\right] \left[\frac{10^2\ cm}{\cancel{10^2}\ cm}\right] \quad \frac{1000}{10^7} \times 100 \times 100$

~~$= 10^7$ earg~~ $\quad 10^7\ \underbrace{g\ cm^2\ s^{-2}}_{erg}$ ~~ergs~~

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Ex:- $G = 6.67 \times 10^{-11}\ Nm^2 kg^{-2}$ (unit u_1)

$u_2 = dyne\ cm^2\ g^{-2}$

$n_2 u_2 = n_1 u_1$

$$\boxed{n_2 = n_1 \frac{u_1}{u_2}}$$

$$n_2 = 6.67 \times 10^{-11} \left[\frac{m^2}{cm^2}\right]\left[\frac{kg^{-2}}{g^{-2}}\right]\left[\frac{N}{dyne}\right]$$

$$= 6.67 \times 10^{-11} \left[\frac{100\,cm}{cm}\right]^{2} \left[\frac{1000\,g}{g}\right]^{-2} \left[\frac{10^5\,dyne}{dyne}\right]$$

$$6.67 \times 10^{-11} \times \frac{10^4}{\cancel{10^6}} \times 10^{-6} \times 10^5$$

~~6.67×10^{-8}~~ $6.67 \times 10^{-11-6+5+4}$

$6.67 \times 10^{-17+9}$

6.67×10^{-8}

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★ Note –

$e^{x, y, z, \alpha}$

- Power = 1
- Power always Number

$E = \frac{3}{2} K \theta$ — Temp [K]

Energy ↵ ↳ Bolt's man constant

$[ML^2T^{-2}]$

$K = \frac{E}{\theta}$ $\left[\frac{ML^2T^{-2}}{K}\right] = [ML^2T^{-2}K^{-1}]$

Pressure, → Volume

$PV = K\,\theta$ — Temp.

↳ Bolt's man constant

$[ML^{-1}T^{-2}L^3] = [ML^2T^{-2}\cancel{K^{-1}}\cancel{K}]$

$[ML^2T^{-2}] = [ML^2T^{-2}]$

$PV = K\theta = E$ $E = h\nu = mc^2$

$P =$

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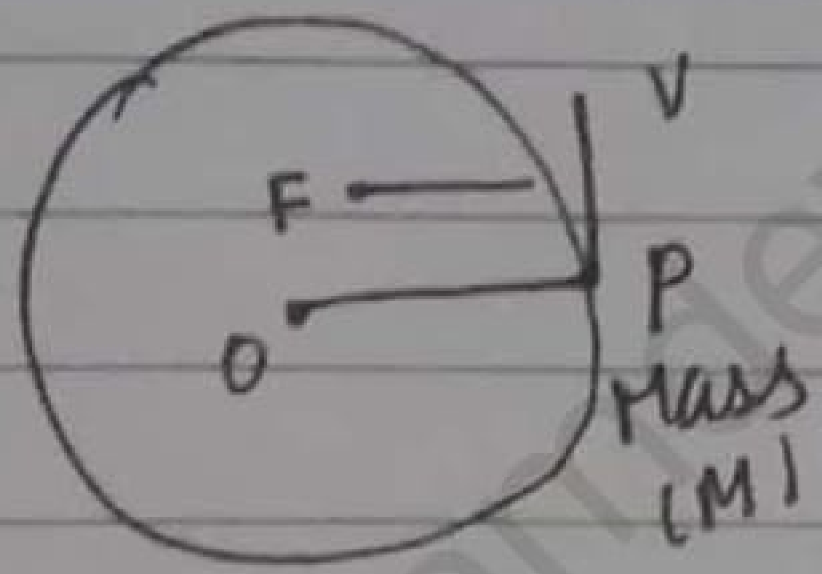
→ To establish relation b/w given physical Quantity

Q1



$OP = R$

$F \to m, v, R$

$F \propto m\, v\, r$

$F = K m v r$

$F = m^a v^b R^c$

Putting dimensional formula for both side

$[MLT^{-2}] = [M]^a [L^1 T^{-1}]^b [L]^c$

$= [M^a L^b T^{-b} L^c]$

$[MLT^{-2}] = [M^a L^{b+c} T^{-b}]$

$a = 1,\ -b = -2,\ b = 2$

$b + c = 1$

$2 + c = 1$

$\boxed{c = 1}$

$F = m^1 v^2 R^{-1}$

$= \frac{Mv^2}{R}$

Ex2:- $F \rightarrow m, r, \omega$

↓ ↓ ↓

Mass radius Angular velocity

$[T^{-1}]$

$[M]$ $[T^{-1}]$

$F \propto m r \omega$

$F = K m r \omega$

$F = m^a r^b \omega^c$ — ①

~~$F = m r \omega^2$~~

$[MLT^{-2}] = [M]^a [L]^b [T]^{-c}$

comparing exponents

$[MLT^{-2}] = [M^a L^b T^{-c}]$ $a = 1$

$b = 1$

$c = 2$

Putting these value in eqn ①

$F = m^1 r^1 \omega^2$

Ex3- $F \rightarrow \eta, r, v$

↓ ↓ ↓

Cofficient of visosity, radius, velocity

$\boxed{K = 6\pi}$

$F = K \eta^a, r^b, v^c$

~~$[M^1 L^{-1} T^{-1}] =$~~

$[MLT^{-2}] = [M^1 L^{-1} T^{-1}]^a [L]^b [LT^{-1}]^c$

$[MLT^{-2}] = [M^a L^{-a} T^{-a} L^b L^c T^{-c}]$

$[MLT^{-1}] = [M^a L^{-a+b+c} T^{-a-c}]$

$\cancel{+}a \cancel{+}c = \cancel{+}2$

$a + c = 2$

$c = 2 - a = 2 - 1 = 1$

$\boxed{c = 1}$

$-a + b + c = b$

$\cancel{-1} + b \cancel{+c} = 1$

$b = 1$

$F = K \eta r v = \boxed{6\pi \eta r v}$

Ex4- $T \rightarrow l\ g$

Time period — Length — acc. due to gravity

$[T]$ $[L]$ $[LT^{-1}]$

$T = k\, l\, g$

$T = l^a g^b$

$[T] = [L]^a [LT^{-2}]^b$

$[M^0 L^0 T] = [L^a] [L^b T^{-2b}]$

$[M^0 L^0 T^{-1}] = [M^0 L^{a+b} T^{-2b}]$

$-2b = 1$

$b = -\frac{1}{2}$ $a = \frac{1}{2}$

$T = 2\pi\, l^{\frac{1}{2}}\, g^{-\frac{1}{2}}$

$T = 2\pi \sqrt{\frac{l}{g}}$

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Imp NCERT $N = -D\left[\frac{n_2 - n_1}{x_2 - x_1}\right]$

D = diffusion cofficient = ?

$n_1\ n_2$ = no. of molecules in unit volume x_1 & x_2

$x_1\ x_2$ = distance [L]

N = No. of molecules passing through per unit Area per unit time $[L^{-2}T^{-1}]$

$$N = -D\frac{\Delta n}{\Delta x}$$

$$D = \frac{N\Delta x}{\Delta n}$$

$$= \left[\frac{L^{-2}T^{-1}L}{L^{-3}}\right]$$

$$= \left[L^{-2+4}T^{-1}\right]$$

$$= \left[L^{+2}T^{-1}\right]$$

(2) $F = \frac{1}{4\pi\varepsilon_0}, \frac{q_1 q_2}{R^2}$

$q_1 q_2$ — charge [A T]

R^2 → distance [L]

F ↓ Force $[MLT^{-2}]$

$4\pi\varepsilon_0$ ↓ ε_0 Electric permitivity

↓ Dimension Less

$\varepsilon_0 = ?$ $\varepsilon_0 = \frac{q_1 q_2}{F R^2} = \frac{[AT][AT]}{[MLT^{-2}][L^2]}$

$[M^{-1} L^{-3} T^4 A^2]$

$F = \mu_0 \frac{I^2 l^2}{r^2}$

I → electric current
l^2 → length
r → Radius
μ_0 → Magnetic permibility

$\mu_0 = \frac{F r^2}{I^2 l^2}$

$= \frac{[MLT^{-2}][L^2]}{[A^2][L^2]}$

$= [M L T^{-2} A^{-2}]$

→ A P silent not $[\bar{1} \bar{3} 4 2]$

ε_0 - Electric permittivity

μ_0 - Magnetic permibility $[1 1 \bar{2} \bar{2}]$
\ Mu mot

$\frac{1}{\sqrt{\mu_0 \varepsilon_0}} = c$
Speed of light

$\sqrt{\frac{\mu_0}{\varepsilon_0}} = R$ → Ω
Resistance.

$R = \frac{V}{I} = \frac{W}{qI}$

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→ Significant figures
The number of digits, which are known reliably in our measurement and one digit that is uncertain are termed as significant figures

→ Rules to determine the numbers of significant figures:-

1. All non-zero digits are significant. 235.75 has five significant figures.
2. All zeroes between two non-zero digits are significant. 2016.008 has seven significant figures
3. All zeroes occuring between the decimal point and the non-zero digits are not significant. provided there is only a zero to left of the decimal point. 0.00652 has three significant figures.
4. All zeroes written to the right of a non-zero digit in a number written without a decimal point are not significant. This rule does not work if zero is a result of measurement. 54000 has two significant figures wheras 54000m has five significant figures.
5. All zeroes occuring to the right of a non-zero digit in a

Seen

1. The significant figures depend upon the least count of the instrument.
2. The no. of significant figure does not depend on the units chosen.
3. In the addition & subtraction operation, the result contains the minimum

number of decimal places of the figures being used.

4. In the multiplication and division operation, the result contains the minimum number of significant ~~value~~ figures.
5. Pure number or unmeasured value do not have significant numbers.
6. Change in the position of decimal does not change the number of significant figures.
   Similarly the change in the units of measured value does not change the significant figures.

→ Rounding off

1. If digit to be dropped is less than 5 then preceding digit should be left unchanged.
2. If digit to be dropped is more than 5 then one should raise preceding digit by one.
3. If the digit to be dropped is 5 followed by a digit other than zero then the preceding digit is increased by one
4. If the digit to be dropped is 5 then the preceding digit is not changed if it is even.
5. If the digit to be dropped is 5 then the preceding digit is increased by one if it is odd.